

What do I need to be able to do?

You should be able to:

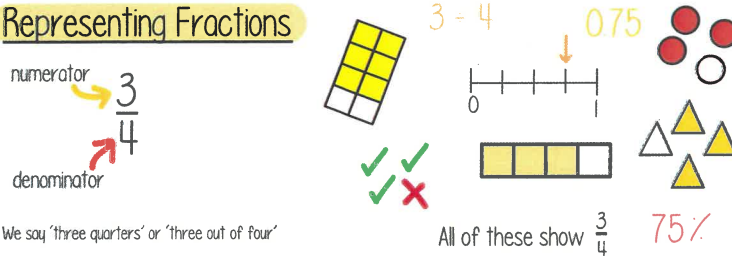
- Understand different representations of fractions
- Fully simplify fractions
- Recognise and find equivalent fractions
- Convert between mixed numbers and improper fractions
- Add/subtract any fractions
- Add/subtract mixed numbers

FRACTIONS

Key Words

- **Numerator:** the top number of a fraction
- **Denominator:** the bottom number of a fraction
- **Equivalent:** of equal value
- **Mixed Number:** a number with an integer and a proper fraction
- **Improper Fraction:** a fraction where the numerator is larger than the denominator
- **Coprime:** two numbers which share no common factors (except 1)

Representing Fractions



Equivalent Fractions

Two fractions are equivalent if they represent the same quantity

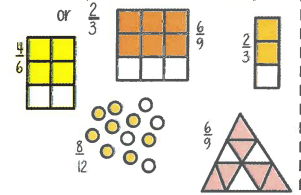
If the numerator and denominator have the same multiplier, they are equivalent

$$\frac{5}{7} \xrightarrow{\times 5} \frac{25}{35}$$

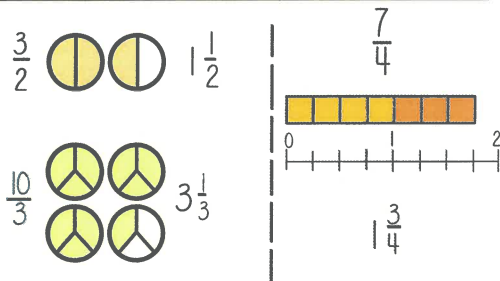
$$\frac{1}{4} \xrightarrow{\times 2} \frac{2}{8}$$

Each of these diagrams represents an equivalent amount.

They all show '2 out of every 3'



Mixed Numbers and Improper Fractions



Fractions can represent more than one whole.

The denominator tells us how many parts make up one whole.

$\frac{9}{5}$ This tells us that one whole is made up of 5 parts. We have 9 parts, so we can make one whole plus 4 parts.

$1\frac{4}{5}$

Simplifying Fractions

You must always simplify your fractions if you can

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{4}{12} = \frac{1}{3}$$

$$\frac{4}{12} = \frac{1}{3}$$

Sometimes a picture can help to visualise the problem

Once you cannot find a common factor, the fraction is fully simplified

$$\frac{7}{10}$$

This fraction is fully simplified as 7 and 10 have no common factors. We can say that 7 and 10 are COPRIME

Adding/Subtracting Fractions

Common denominators

$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$

$\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$

$\frac{1}{10} + \frac{3}{10} = \frac{4}{10} = \frac{2}{5}$

Remember that the denominator doesn't change

We can just subtract 4 from 5

4 and 10 have a common factor (2)

You must always fully simplify your fractions

Adding/Subtracting Fractions

Common multiples

$\frac{3}{5} + \frac{1}{10} = \frac{7}{10}$

10 is a multiple of 5 (5 x 2) so, using equivalent fractions we can say: $\frac{3}{5} = \frac{6}{10}$

$\frac{6}{10} + \frac{1}{10} = \frac{7}{10}$

$\frac{3}{4} - \frac{1}{12} = \frac{5}{6}$

12 is a multiple of 4 (4 x 3) so, using equivalent fractions we can say: $\frac{3}{4} = \frac{9}{12}$

$\frac{9}{12} - \frac{1}{12} = \frac{8}{12} = \frac{2}{3}$

Remember you must always fully simplify your fractions!

$\frac{1}{2} + \frac{2}{3} + \frac{1}{6} = \frac{4}{2} = 2$

Here, we know that 2 and 3 have a common multiple of 6, so we can say: $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$

$\frac{3}{6} + \frac{4}{6} + \frac{1}{6} = \frac{8}{6} = \frac{4}{3} = 1\frac{1}{3}$

We need to give our answer as a mixed number

Adding/Subtracting Fractions

Different denominators

$\frac{1}{5} + \frac{3}{4} = \frac{7}{20}$

We need to find a common denominator using equivalent fractions

$\frac{1}{5} = \frac{4}{20}$

$\frac{3}{4} = \frac{15}{20}$

$\frac{4}{20} + \frac{15}{20} = \frac{19}{20}$

$\frac{3}{11} + \frac{2}{3} = \frac{9}{33} + \frac{22}{33} = \frac{31}{33}$

The LCM of 11 and 3 is 33, so our equivalent fractions are:

$\frac{3}{11} = \frac{9}{33}$

$\frac{2}{3} = \frac{22}{33}$

$\frac{5}{7} + \frac{4}{9} = \frac{45}{63} + \frac{28}{63} = \frac{73}{63}$

Remember you can find the LCM of 7 and 9 by listing their multiples: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119, 126, 133, 140, 147, 154, 161, 168, 175, 182, 189, 196, 203, 210, 217, 224, 231, 238, 245, 252, 259, 266, 273, 280, 287, 294, 301, 308, 315, 322, 329, 336, 343, 350, 357, 364, 371, 378, 385, 392, 399, 406, 413, 420, 427, 434, 441, 448, 455, 462, 469, 476, 483, 490, 497, 504, 511, 518, 525, 532, 539, 546, 553, 560, 567, 574, 581, 588, 595, 602, 609, 616, 623, 630, 637, 644, 651, 658, 665, 672, 679, 686, 693, 700, 707, 714, 721, 728, 735, 742, 749, 756, 763, 770, 777, 784, 791, 798, 805, 812, 819, 826, 833, 840, 847, 854, 861, 868, 875, 882, 889, 896, 903, 910, 917, 924, 931, 938, 945, 952, 959, 966, 973, 980, 987, 994, 1001, 1008, 1015, 1022, 1029, 1036, 1043, 1050, 1057, 1064, 1071, 1078, 1085, 1092, 1099, 1106, 1113, 1120, 1127, 1134, 1141, 1148, 1155, 1162, 1169, 1176, 1183, 1190, 1197, 1204, 1211, 1218, 1225, 1232, 1239, 1246, 1253, 1260, 1267, 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10493, 10500, 10507, 10514, 10521, 10528, 10535, 10542, 10549, 10556, 10563, 10570, 10577, 10584, 10591, 10598, 10605, 10612, 10619, 10626, 10633, 10640, 10647, 10654, 10661, 10668, 10675, 10682, 10689, 10696, 10703, 10710, 10717, 10724, 10731, 10738, 10745, 10752, 10759, 10766, 10773, 10780, 10787, 10

What do I need to be able to do?

You should be able to:

- Multiply unit fractions
- Multiply non-unit fractions
- Use cross-cancelling to simplify fractions before multiplying
- Divide integers by fractions
- Divide fractions by fractions
- Find fractions of amounts
- Use a given fraction to find the whole
- Find the reciprocal of an integer/fraction

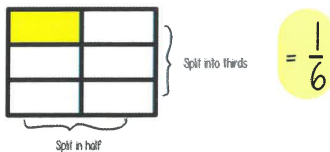
FRACTIONS 2

Key Words

- Numerator: the top number of a fraction
- Denominator: the bottom number of a fraction
- Unit fraction: a fraction with a numerator of one
- Commutative: changing the order of the operations doesn't change the result
- Reciprocal: the reciprocal of a number is 1 divided by the number
- Coprime: two numbers which share no common factors (except 1)

Multiplying unit fractions

$$\frac{1}{2} \times \frac{1}{3} \quad \text{"One half of one third"}$$

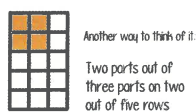


Multiplying any fractions

Example 1

$$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$$

"Two thirds of two fifths"



Example 2

$$\frac{5}{7} \times \frac{14}{15} = \frac{5 \times 14}{7 \times 15} = \frac{70}{105}$$

Remember to simplify where possible = $\frac{2}{3}$

Example 3

$$\frac{1}{2} \times 2\frac{1}{3} = \frac{1}{2} \times \frac{7}{3} = \frac{7}{6} = 1\frac{1}{6}$$

See cross-cancelling for a quicker method

Cross Cancelling Method

$$\frac{2}{3} \times \frac{6}{7} = \frac{4}{7}$$

1 and 3 both have a common factor of 3, so we can divide both by 3.

The method means that we do not need to simplify our answer as it should be fully simplified already!

Example 1

$$\frac{5}{9} \times \frac{18}{25} = \frac{2}{5}$$

this becomes

$$\frac{1}{1} \times \frac{2}{5} = \frac{2}{5}$$

Remember:
Multiply the numerators then multiply the denominators

Example 2

$$\frac{15}{27} \times \frac{36}{45} = \frac{4}{9}$$

this becomes

$$\frac{1 \times 4}{3 \times 3} = \frac{4}{9}$$

Dividing integers by a unit fraction

$$3 \div \frac{1}{3} \quad \text{Think of this as 'how many times does a third go into 3?'}$$



there are three thirds in one whole, so there are 9 thirds in 3 wholes

Reciprocals

A number multiplied by its reciprocal is always 1

$$2 \times \frac{1}{2} = 1$$

$$5 \times \frac{1}{5} = 1$$

The reciprocal of a is $\frac{1}{a}$

Dividing by a fraction, $\frac{1}{a}$, is the same as multiplying by its' reciprocal, a.

Example:

$$3 \div \frac{1}{3} = 9$$

$$3 \times 3 = 9$$

Dividing Fractions

Example 1

$$\frac{2}{3} \div \frac{5}{7}$$

$$\frac{2}{3} \times \frac{7}{5}$$

$$\frac{2 \times 7}{3 \times 5}$$

$$\frac{14}{15}$$

Example 2

$$\frac{5}{12} \div \frac{25}{18}$$

$$\frac{1}{2} \times \frac{18}{25}$$

$$\frac{1 \times 3}{2 \times 5}$$

$$\frac{3}{10}$$

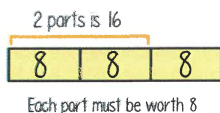
Finding Fractions of Amounts

Find $\frac{1}{2}$ of 10 "Share 10 into 2 equal parts"



Find $\frac{2}{3}$ of 24

$$\frac{1}{3} \text{ is } 8 \text{ as } 24 \div 3 = 8$$



$$\frac{2}{3} \text{ of } 24 = 16$$

Find $\frac{7}{10}$ of £105

$$£105 \div 10 = £10.50$$



$$\frac{7}{10} \text{ of } £105 = £73.50$$

Reverse Fractions of Amounts

$\frac{3}{4}$ of a number is 15. What is the number?

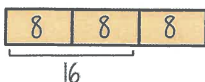


15

If 3 parts = 15, then one part must = 5

The original number was 20

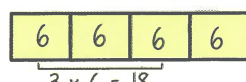
$\frac{2}{3}$ of a number is 16. What is $\frac{3}{4}$ of the number?



16

If 2 parts = 16, then one part must = 8

The number is 8 x 3 = 24. So what is $\frac{3}{4}$ of 24?

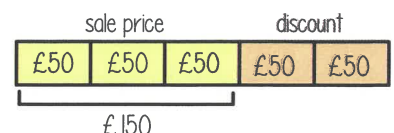


One quarter is 6

$$= 18$$

Worded problem

A TV is on sale for $\frac{2}{5}$ off the price. It now costs £150. How much did it cost originally?



So the original price of the TV was $5 \times £50 = £250$