Knowledge Organiser: Algebra – the basics

What you need to know:

Collecting like terms

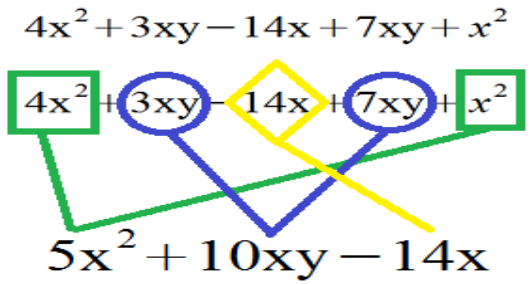
Simplify the expression: **4w + 3 + 2w - 1**

$4w + 3 + 2w - 1$ (Now Group Like Terms)

$= 4w + 2w + 3 - 1$ (Combine Like Terms)

$= 6w + 2$

$= 6w + 2$ ✓



Note – you can only collect terms that have the same power eg $5x + 4x^2 \neq 9x^2$

Substitution

Evaluate (find the value of) the expressions, given that:

$a = 2, \quad b = 3, \quad c = -5$

Note – Always use the correct order of operations

1. $4b = 4 \times 2 = 8$
2. $7b - 3c = (7 \times 3) - (3 \times -5) = 21 - -15 = 21 + 15 = 36$
3. $5b^2 + 1 = 5 \times (3)^2 + 1 = 5 \times 9 + 1 = 45 + 1 = 46$
4. $2c^3 = 2 \times (-5)^3 = 2 \times -125 = -250$
5. $\frac{3ac}{2b} = \frac{3 \times 2 \times -5}{2 \times 3} = \frac{-30}{6} = -5$

For fractions work out the numerator and denominator separately first

Key Terms:

Formula: expresses the relationship between two or more unknown values

Expression: A sentence in algebra that does NOT have an equals sign

Identity: One side is the equivalent to the other side

Substitution: Replace the letter with a given value

Like terms: Variables that are the same are 'like'

Expand: Single brackets – each term inside the bracket is multiplied by the term outside the bracket.
Double brackets – each term in the first bracket is multiplied by all the terms in the second bracket.

Factorise: Putting an expression back into brackets

You need to be able to:

- Identify an expression/equation/formula/identity from a list
- Manipulate and simplify algebraic expressions by collecting 'like' terms
- Substitute numbers into formulae
- Simplify expressions
- Use index notation and the index laws
- Multiply a single term over a bracket and simplify by factorising
- Expand double brackets
- Factorise quadratic expressions

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Collecting like terms 156-157
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Expand single brackets 160-161
Factorise single brackets 168-169
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Knowledge Organiser: Algebra – The Basics

What you need to know:

Linear expressions

Expand and simplify where appropriate

1) 7 (3 + a) = 21 + 7a

2) 2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a
= 5a + 16

Note – collect like terms to simplify

3) Factorise $9x + 18$ = $9(x + 2)$

4) Factorise $6e^2 - 3e$ = $3e(2e - 1)$

Note – to 'factorise fully' means take out the HCF.

Quadratic expressions

Expand and simplify:

1) $(p + 2)(2p - 1)$
$= 2p^2 + 4p - p - 2$
$= 2p^2 + 3p - 2$

2) $(p + 2)^2$
$(p + 2)(p + 2)$
$= p^2 + 2p + 2p + 4$
$= p^2 + 4p + 4$

Factorise:

3) $x^2 - 2x - 3$
$= (x - 3)(x + 1)$

Factorise and solve:

4) $x^2 + 4x - 5 = 0$
$(x - 1)(x + 5) = 0$
Therefore the solutions are:
Either $(x - 1) = 0$
$x = 1$
Or $(x + 5) = 0$
$x = -5$

Index Laws

Simplify the following: $a^3 \times a^4$

If we start by writing it out in full:

$a^3 = a \times a \times a$

$a^4 = a \times a \times a \times a$

$\therefore a \times a \times a \times a \times a \times a \times a = a^7$

To multiply together two identical values or variables (letters) that are presented in index form, add the powers.

$d^8 \times d^2 = d^{10}$ $e^{-3} \times e^5 = e^2$ $f \times f^3 = f^4$

Simplify the following:

$\frac{m^5}{m^3}$

If we start by writing it out in full:

$m^5 = m \times m \times m \times m \times m$

$m^3 = m \times m \times m \times m \times m$

$\therefore \frac{m \times m \times m \times m \times m}{m \times m \times m} = m^2$

$p^6 \div p^3 = p^3$ $\frac{s^3}{s^7} = s^{-4}$

To divide two identical values or variables (letters) that are presented in index form, subtract the powers

Simplify the following:

$(m^7)^3 = m^{21}$ $(n^6)^{-4} = n^{-24}$

To raise a value or variable (letter) presented in index form to another index, multiply the powers together

Reminders:

$a^1 = a$

$a^0 = 1$

Difference of two squares

$a^2 - b^2 = (a + b)(a - b)$

Index Laws - Negative

$a^{\frac{1}{2}} = \sqrt{a}$ $a^{-m} = \frac{1}{a^m}$

$9^{\frac{1}{2}} = \sqrt{9} = 3 \text{ or } -3$

$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

$7^{-1} = \frac{1}{7}$

$8^{-4} = \frac{1}{8^4} = \frac{1}{4096}$

Knowledge Organiser: Algebra – The Basics (solving equations)

What you need to know:

Solving Equations

Unknown on one side

Solve 2x + 1 = 9

-1 -1

2x = 8

÷2 ÷2

x = 4

Solve 3(y – 7) = 9

Always expand the bracket first

3y – 21 = 9

+21 +21

3y = 30

÷3 ÷3

y = 10

You can check your answers by substituting your answer back into the question

Unknowns on both side

Solve 2d - 7 = 5d - 10

Start by subtracting the smallest amount of the variable from both sides

-2d -2d

-7 = 3d - 10

+10 +10

3 = 3d

÷3 ÷3

d = 1

Solve 3(2t + 4) = 2(2 - t)

6t + 12 = 4 – 2t

+2t +2t

8t + 12 = 4

-12 -12

8t = -8

÷8 ÷8

t = -1

Set up equations from word problems

Jenny, Kenny, and Penny together have 51 marbles. Kenny has double as many marbles as Jenny has, and Penny has 12. How many does Jenny have?

Set up an equation then solve

Jenny's + Kenny's + Penny's = 51

n + 2n + 12 = 51

3n + 12 = 51

-12 -12

3n = 39

÷3 ÷3

n = 13

Key Terms:

Solve: Find a numerical value that satisfies the equation

Inverse operation: The operation that reverses the effect of another operation e.g. subtraction in the inverse of addition

Rearrange: Change the order of the terms

You need to be able to:

- Solve linear equations
- Derive a formula and set up equations from worded problems
- Change the subject of a formula
- Use iteration to find approximate solutions to equations

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Solving equations 176-188

Rearranging formulae 279-287

Iteration 322

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Knowledge Organiser: Algebra – The Basics (solving equations)

What you need to know:

Rearranging Formulae

Change the order of the terms so 'u' is on its own

Make u the subject: $v = u + at$

-at -at

$v - at = u$

so $u = v - at$

Make m the subject: $I = mv - mu$

If the letter appears twice you will need to factorise

÷ (v – u) ÷ (v – u)

$I = m(v - u)$

$I \div (v - u) = m$

$$m = \frac{I}{v - u}$$

Iteration

Starting with $x_0 = 0$ use the iteration formula

$$x_{n+1} = \frac{2}{x_n^2 + 3}$$

3 times to find an estimate to the solution.

Calculate the values of x_1, x_2, x_3 to find an estimate for the solution to $x^3 + 3x = 2$

$$x_{0+1} = \frac{2}{0^2 + 3} = 0.\dot{6}$$

We substitute this value into the next step.

$$x_{1+1} = \frac{2}{0.\dot{6}^2 + 3} = 0.5806451613$$

$$x_{2+1} = \frac{2}{(0.58\ldots)^2 + 3} = 0.5993140006$$

An estimate of the solution is 0.6 because all of the solutions round to 1d.p.

Key Concepts

Iteration is the **repetition** of a mathematical procedure applied to the result of a previous application, typically as a means of **obtaining successively closer approximations** to the solution of a problem.

Equations of motion

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

s – displacement
u – initial velocity
v – final velocity
a – acceleration
t - time

You will be asked to use these equations and substitute into them. You do not need to memorise them.